Measuring Profit in Cooperatives: Definition and Methods

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Abstract

The purpose of this study was to present an overview of measuring profits in different types of cooperatives using a classification scheme provided by the National Cooperative Business Association (NCBA). Thecooperative form of business structure was defined for the eleven categories of cooperatives in the NCBA classification. This was followed by the delineation of the theoretical economic basis and nonlinear constrained optimization model for each category. We find that profit formulations from the literature are strictly utilitarian with the achievement of economies of scale in purchasing to reduce costs for members to at or near the level of marginal revenue equating marginal cost, or selling to provide price supports such as in agricultural cooperatives whereby market power may strengthened through the restriction of quantity. This study identifies another type of model with sharing of knowledge and skills as occurs with preschool cooperatives among parents or technology cooperatives among programmers and software developers.

Keywords: cooperatives, profitability, nonlinear optimization, agricultural cooperative, cooperative banking

1. Introduction

The fundamental financial dichotomy between the cooperative form of organization and for-profit firms lies in the treatment of net income. In a for-profit firm, stockholders purchase stock. Their investment is used to produce goods and services, which are sold for revenue. After payment of expenses, a percentage of net income is returned to shareholders as dividends while the balance is reinvested as retained earnings with the objective of increasing stock prices. The motivation of business existence is to achieve higher profits and in turn, generate positive stock returns. With cooperatives, members are the owners, making an initial investment which may be used to either purchase member output (such as a dairy farmer's output of milk) for resale at higher prices through a system of price supports, or negotiate lower prices for services needed by members. The purpose of this study isto present an overview of measuring profits in different types of cooperatives.

2. Review of Literature

The literature offers a limited number of formulations of profit in cooperatives so that we may firstevaluate studies of for-profit firms with collaborative arrangements to promote market power. Dechenaux and Kovenock (2007) construct a model of tacit collusion which tentatively achieves equilibrium (though this equilibrium lacks the stability of the cooperative) in which selling occurs at a uniform price with the assumption of full satisfaction of demand. In their second model, participants are assumed to have residual demand with each participant submitting prices that are successively higher, with the understanding that the submitted price is below that which may be obtained from the open market.

The authors recommended the selection of the highest submitted price as the collaboration's price, with purchases from the lowest-priced producer, thenthe next lowest-price producerandso on until the smallest quantity is purchased from the highest priced producer.

While this model provides a substantial financial benefit to lower-priced producers, its benefit to the highest-priced producer is unclear at this individual has no incentive to remain within the collaborative agreement and can simply withdraw. Murray (1995) observed oligopolistic behavior among pulpwood and sawmill producers on the basis of differential transportation costs. As transportation costs are high and vary among wood producers, large wood suppliers with access to preferential transportation rates collaborated to set lower purchase prices to mills. This collaboration was short-lived, particularly in the saw log market, where smaller producers found the means to transport wood cheaply, thus driving saw logs to a perfectly competitive market. Paper mills, i.e. the customers of the factor inputs including pulpwood and saw logs become oligopolistic over subsequent decades with increasing concentration (four large paper mills dominate the market) with the power to set the lowest purchase prices to the highly competitive and fragmented wood producers.

For a purchasing cooperative, Marini and Zevi (2011) constructed a profit function in which members of a retail cooperative accepted the average production cost as the cooperative price to be paid to wholesalers for a limited quantity of differentiated goods. Any surplus earned by the cooperative was refunded to customers in proportion to their total production spending. The cooperative price forced for-profit-retailers to charge reduced prices to wholesalers, thereby improving market efficiency. We have some reservations about this conclusion. Given capacity restrictions on production, a cooperative can only purchase a limited quantity of factor inputs to sell a limited amount of output. The remainder of factor inputs will be produced by for profit firms who face no competition from the cooperative, and consequently charge unrestricted market prices. We consider this sequence of actions to be more realistic, without any improvement in market efficiency. Anderson et al. (1980) shed more light on the effect of price changes of factor inputs in purchasing cooperatives. If a factor input is a normal good and its price increases, the cost of production will increase, and membership will have to increase so that the larger quantity being produced will maintain price at the minimum of long-run average cost. On the other hand, if the factor good is an inferior good, price increases will leave the cost of production unaffected as the quantity produced will decrease (the elasticity of marginal cost will determine if the cost of production will decrease in proportion to the increase in unit price).

3. Models of Cooperatives

3.1. Agricultural Cooperatives

Milk is sold in a perfectly competitive market. Market power is limited for the individual dairy farmer as thousands of alternate milk producers are present. In repeated negotiations or continuous auctions, a uniform price is set provided all members adhere to capacity constraints. The market consumes all of the quantity supplied at the uniform price. The perfectly competitive market is transformed into an oligopoly with each individual milk producer receiving an equal share of the profit.Suppose the quantity demanded for an individual milk producer's milk is $q_{s,}$ to be sold at optimal price $p_{s,,}$set by the individual producer. The dairy farmer is unable to sell at full capacity, i.e. has unsold milk, or residual demand for unsold milk for all farmers> 0. The cooperative seeks to eliminate residual demand by restricting individual excess milk supply to $q_{i,.}$To maintain the capacity constraint, Farmer 1 will sell a certain quantity $q_1$through the cooperative. This quantity will be observed by Farmer 2, who will attempt to sell a larger quantity. Farmer 1 will increase his or her original quantity and so on in several rounds of quantity observation and quantity matching. They will finally converge to a Cournot equilibrium in which part of the output of each farmer will be sold at q_1 and q_2 respectively.

From Etro (2006),
p_1 = Farmer 1's price, p_2 = Farmer 2's price, q_1 = Farmer 1's quantity, q_2 = Farmer 2's quantity, c = marginal cost for each farmer Equilibrium prices will be:

$p_1 = p_2 = P(q_1 + q_2)$(1)

This implies that Farmer 1's profit is given by

$\Pi_1 = q_1(P(q_1 + q_2) - c)$. (2)

If Farmer 1 believes Farmer 2 is producing quantity q_2 , graphically,2 points are needed to determine Farmer 1'sresidual demand. If Farmer 1 decides not to produce, Farmer 2's price is given by $P(0 + q_2) = P(q_2)$. If Farmer 1 produces q_1' then Farmer 2's price is given by $P(q_1' + q_2)$.

In the general case, for each quantity that farmer 2 might decide to set, the corresponding price is given by a downward sloping demand curve, $d_1(q_2)$. Farmer 1's optimum output is at the point at which marginal revenueequals marginal cost. Marginal cost (c) is assumed to be constant. Marginal revenue is a curve - $r_1(q_2)$ - with twice the slope of $d_1(q_2)$ and with the same vertical intercept. The point at which the two curves (c and $r_1(q_2)$) intersect corresponds to Farmer 1's optimal quantity$q_1''(q_2)$. Farmer 1's optimum $q_1''(q_2)$, depends on what he or she believes Farmer 2 is doing. To find an equilibrium, we derive Farmer 1's optimum for other possible values of q_2 .If $q_2 = 0$, then the first farmer's residual demand is effectively the market demand, $d_1(0) = D$. The optimal solution is for Farmer 1 to choose the monopoly quantity; $q_1''(0) = q^m$ (q^m is monopoly quantity). If Farmer 2 were to choose the quantity corresponding to perfect competition, $q_2 = q^c$ such that $P(q^c) = c$, then Farmer 1's optimum would be to produce 0.$q_1''(q^c) = 0$. The function $q_1''(q_2)$ is firm 1's reaction function, as it gives firm 1's optimal choice for each possible choice by firm 2. The last stage in finding the Cournotequilibrium is to find Farmer 2's reaction function. In this case, it is symmetrical to Farmer 1's as they have the same cost function. The equilibrium is the intersection point of the reaction curves, $R_1(q_2)$ and $R_2(q_1)$. This intersection occurs at Nash equilibrium output levels, which are given by the following expressions.Dairy farmer 1's optimal quantity is

$$q_1 = \frac{a - q_2 - \partial C_1(q_1) / \partial q_1}{2} \quad (3)$$

It follows that dairy farmer 2's optimal quantity is:

$$q_2 = \frac{a - q_1 - \partial C_2(q_2) / \partial q_2}{2} \quad (4)$$

For the cooperative, the total optimal quantity to be sold to milk processing plants,

$$Q_i = (q_1 + q_2 + q_3 \ldots\ldots q_n) \quad (5)$$

The price charged to milk processing plants = weighted average price in proportion to the quantity of milk sold to the cooperative = $[(p_1q_1/Q_i) + (p_2q_2/Q_i) + \ldots.(p_nq_n/Q_i)]$

$$Surplus = [(p_1q_1/Q_i) + (p_2q_2/Q_i) + \ldots.(p_nq_n/Q_i)]-C \qquad (6)$$

Surplus = Return on capital to members, Π *to cooperative = 0,*

C = total cost of production of the cooperative.

3.2. Child Care Cooperatives

Preschool rates range from \$4,460 to \$13,158 per year (\$372 to \$1,100 monthly), according to the National Association of Child Care Resource & Referral Agencies (NACCRRA, 2012). In contrast, cooperative preschools cost less, but defray costs by requiring parents or other family members to undertake classroom responsibilities.

We adapt the Bertrand (1883) formulation,

MC = constant marginal cost,
p_1 = cooperative firm's price,
p_2 = for-profit firm's price,
p_m = monopoly price,
$c_1 = c_2$ =unit cost of production,
Suppose both firms price their services equally just above marginal cost,

or $p_1 = p_2 > MC$, or $(p_1 + \Delta) - MC = 0$, or $(p_2 + \Delta) - MC = 0$,(7)
each firm would capture half of the market. There is an incentive in successive rounds of trading to reduce prices to the marginal cost level, which becomes the final price. Either firm which reduces prices above marginal cost faces retaliation by the other till both firms reduce prices to the marginal cost level.

On a graph of p_2 versus p_1, the marginal cost level of pricing will be depicted as $p_1"(p_2)$ or $p_2"(p_1)$ which corresponds to the monopoly price, p_m at any point above marginal cost.

For the cooperative,

$\Pi =[p_1"(p_2(q_1)-c_1q_1]$(8)

Π = *surplus return on capital to members, Π to cooperative = 0.*

3.3. Credit Unions

Credit unions receive funds from depositors, pay them interest, and loan those funds to members at lower rates than banks. We envision a credit union offering deposit rates of $S_s$to Q_s members in the following production function, and offering financial planning services,

$F_pC = S_s Q_s + F_p$(9)

while receiving interest on non-credit card loans and credit card revenue thus,

$R = I_sQ_r + R_i$(10)

Where R = revenue, I_s = interest rate on non-credit card loans, and credit card interest and fees

Q_r = number of borrowers,

R_i = revenue from sale of insurance products.

As both revenue from loans and expense from deposits depend upon market interest rates, we apply the expectations theory of interest rates, whereby both the rate of change in revenue and the rate of change in expensevary with the nominal interest rate, and in turn, the number of borrowers and lenders.

$\partial C/\partial IN\ \partial IN/.\partial S_s = Q_s$

$\partial R/\partial IN\ \partial IN/.\partial I_s = Q_r$

IN = nominal interest rate. We assume that the utility of loan demand and deposit supply is strictly concave, the profit function which maximizes loan interest revenue.For a cooperative, there is a minimum profit constraint,where the surplus S_x is the minimum profit with λ and μ as

Kuhn-Tucker multipliers,

$Min = -(I_sQ_r + R_i)$(11)

Subject to

$S_x \leq -(I_sQ_r + R_i)-(S_s Q_s + F_p)$

$Q_r \geq 0$

$Q_s \geq 0$

First-order conditions include:

$d/dQ_r(I_sQ_r+R_i)(1+\mu)-\mu\ d/dQ_s(S_sQ_s+F_p) \leq 0$(12)

$Q_r \geq 0$

$Q_s \geq 0$

$Q_r[(I_sQ_r+R_i)(1+\mu)-\mu d/dQ_s(S_sQ_s+F_p)] = 0$(13)

$(I_sQ_r+R_i)-(S_sQ_s+F_p)-S_x \geq 0$

$\mu \geq 0,$

$\mu[(I_sQ_r+R_i)-(S_sQ_s+F_p)-S_x] = 0$

3.4. Cooperative Banks

Cooperative banks fund projects that would not be cost-effective for a for-profit bank, such as meeting the initial investment for a rural utility. Rural utilities have to place power lines and telephone lines over long distances and inaccessible terrain increasing the cost of initial investment which may have to be recovered over a longer period of time.If the initial investment is I_t, annual cash flows from operations are *CF*and the payback period is *n,* where $n>t$,

$I_t = CF_{t+1}/(1+r)^{t+1} + CF_{t+2}/(1+r)^{t+2} + CF_{t+3}/(1+r)^{t+3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots CF_{t+n}/(1+r)^{t+n}$(14)

If a cooperative bank loans the amount of the initial investment to a rural utility, the bank must assure that the above cash flows will be received to meet loan payments. Accordingly, the bank (which manages the utilities'investments) may supplement loan interest revenue (*LI*) and consumer utility receipts (*CU*) with investments in highly rated bonds with varying maturities so that a steady stream of cash flows from bond interest revenue (*BI*)and return of par values at maturity (*Pa*) will ensue, in that as one bond is retired another will take its place to maintain continuity of bond interest and maturity values. If *Q* is the number of utility customers,

$CF_{t+1} = [(LI_t + CU_tQ_t + BI_t + Pa_{t+1}/(1+r)^{t+1}) + (LI_{t-1} + CU_{t-1}Q_{t-1} + BI_{t-1} + Pa_{t+1}/(1+r)^{t+1})$
$+\ldots(LI_{t-n} + CU_{t-n}Q_{t-n}BI_{t-n} + Pa_{t+1}/(1+r)^{t+1})]$(15)

Additionally, the bank may undertake the bill paying and utility bill collection responsibilities of the utility to improve the efficiency of day-to-day operations. Assuming that consumer utility bill deposits and bill payments follow a Poisson distribution with arrival rate, λ and service rate, μ in a single queue with multiple bank tellers, the bank must seek to minimize its average response time$1/(\mu - \lambda)$to improve efficiency. This arrival rate and service rate are distinct from the , λ and μ listed below, which are Kuhn-Tucker multipliers.

Minimize $-[CF_t + 1/(\mu - \lambda)]$

Subject to

$S_x \leq (CF_t - T - IN)$

$Q \geq 0$

where,

T = taxes

IN = Interest expense

S = surplus

First-order conditions include:

$d/dQ(CF_t)(1+\mu)-\mu\ d/dQ(T\text{-}IN) \leq 0$ (12)

$Q \geq 0$

$Q[(CF_t)(1+\mu)-\mu d/dQ_s(T\text{-}IN)] = 0$ (13)

$(CF_t)\text{-}(T\text{-}IN)\text{-}S_x \geq 0$

$\mu \geq 0,$

$\mu[(CF_t)\text{-}(T\text{-}IN)\text{-}S_x] = 0$

3.5. Funeral Home Cooperatives

Nationally, three funeral home cooperatives existed in 2011 with the largest one, People's Memorial Association having 100,000 members in a single state. They offer low-cost funerals ranging in price from $ 849-$ 2,799.Full service funeral homes offer limousine transportation, embalming with cosmetology and restorative art, clergy choices, cemetry choices, grief counseling, family notification, preservation of gifts from viewers and filing for benefits. In a survey of 170 funeral homes in Western and Central Washington averaged $ 1,492 for a cremation to $ 3,946 for a full-service funeral with embalming, viewing, and chapel services.

Consider a family member faced with the decision of the type of funeral to hold.

Min $(c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5) + c_6x_6 + c_7x_7$(15)

St

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 0$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_p = 0,1$

$c_{1,}$ = cost of direct cremation at a cooperative,

c_2 = cost of direct cremation with memorial service at a cooperative,

c_3 = cost of direct burial without graveside service at a cooperative,

c_4 = cost of direct burial with graveside service at a cooperative,

c_5 = cost of full funeral service with basic casket at a cooperative,

c_6 = cost of full funeral service with metal casket at a cooperative,

c_7= cost of a funeral service at a funeral home,

x_1 = a dichotomous variable with values of 1,0of the decision to have or not have a direct cremation at a cooperative,

x_2 = a dichotomous variable with values of 1, 0of the decision to hold or not to hold a direct cremation with memorial service at a cooperative,

x_3 = a dichotomous variable with values of 1, 0, of the decision to hold or not to hold a direct burial without graveside service at a cooperative,

x_4 = a dichotomous variable with values of 1, 0, of the decision to hold or not to hold a direct burial with graveside service at a cooperative,

x_5 = a dichotomous variable with values of 1,0 of the decision to hold or not to hold a full funeral service with basic casket at a cooperative,

x_6 = a dichotomous variable with values of 1,0 of the decision to hold or not to hold a full funeral service with metal casket at a cooperative,

x_7 = a dichotomous variable with values of 1,0of the decision to hold or not to hold a full funeral service at a funeral home,

x_p = exterior penalty to reduce the cost of a funeral by eliminating additional services.

Adapting exterior penalty function methods Avriel (2003) to funerals, a penalty is imposed on every additional service. We assume that the decision-maker begins with the desire to have a traditional funeral. As he or she evaluates the cost of each amenity, eliminating each additional services by imposing penalties successively. Different individuals will cease the process at different stages with varying levels of adoption of additional services.Suppose in the first stage, a graveside metal casket cooperative plan is chosen and the decision-maker chooses to give up a procession (extra service). The procession may be presented as η so that the minimum costfunction at stage 1 is

$\varphi(\eta) = |min(0,\eta)|^{\alpha}$and the procession has the following function, $\zeta(\eta) = |\eta|^{\beta}$

where α and β are given constants, with values of 1 or 2. The exterior penalty function method continues to solve constrained optimizations for k = 0, 1, 2, ….., given by

min $F(x, p^k) = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5) + c_6x_6 + c_7x_7 + 1/p^k\{\sum^{m}_{i=1|}\min[0, x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 0]|^{\alpha} + \sum_{j=1}^{p}| x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_p = 0,1|^{\beta}\}$

If x^{k*} is the optimal choice of funeral, there will be a sequence of points (x^k) which converges to the optimal choice.From the cooperative's perspective, the profit may be defined as,

$\Pi = (MF + FF - CP - W\text{-}CL) - S = 0$ (16)

where

MF = membership fee

FF= funeral fee

CP=cost of production including transportation costs, urns, and caskets

W=wages including benefits

CL= cost of leasing property including buildings and equipment*

S=surplus returned to members operative

3.6. Health Care Cooperatives-Independent Pharmacies

Independent pharmacy cooperatives consist of group purchasing organizations that band together to purchase prescription drugs and other products This collaboration permits them to remain price competitive with large pharmacy chains. The cooperative returns substantial surpluses achieved through cost savings. For example, Independent Pharmacy Cooperative (IPC, 2010)with 4,500 members paid total rebates of over $ 198 million at an unit amount averaging $ 67,000 (IPC, 2010). Prescription drugs may be purchased daily, over-counter-drugs may be discounted to about 15% below the wholesale price, and vendor contracts provide competitive pricing on retail merchandise items.On the other hand, non-cooperative purchased items catering to distinct market segments may be sold at market prices with significant markups including supplies to senior facilities and deliveries of home medical equipment. The profit function may be stated as follows,

Max $(r_1 – c_1)x_1 + (r_2 – c_2)x_2 + (r_3 – c_3)x_3 + (r_4-c_4)x_4 +(r_5 – c_5)x_5 + (r_6-c_6)x_6$(17)

St

$x_1 \geq M$

$x_2 \geq N$

$x_3 \geq O$

where

$r_1, r_2, r_3, r_4, r_5, r_6$ = revenue per unit of prescription drugs, over-the-counter drugs, retail merchandise, immunizations ,supplies to senior facilities, and home medical equipment respectively,

$c_1, c_2, c_3, c_4, c_5, c_6$ = cost per unit of prescription drugs, over-the-counter drugs, retail merchandise, immunizations ,supplies to senior facilities, and home medical equipment respectively,

M, N, O= minimum volume of purchases of prescription drugs, over-the-counter drugs and retailmerchandise respectively for the cooperative to qualify for volume discounts on purchases

Assumingthat each independent pharmacy has a binding upper bound on the specialized products that it may sell (a pharmacy can only sell a limited number of prosthetic shoes, for example),

Min $–[(r_1 – c_1)x_1 + (r_2 – c_2)x_2 + (r_3 – c_3)x_3 (r_4-c_4)x_4 +(r_5 – c_5)x_5 + (r_6-c_6)x_6]$(18)

St

$x_1 -M \geq 0$

$x_2 -N\geq 0$

$x_3-O\geq 0$

$x_5 – Q = 0$

$x_6– R = 0$

where,

Q,*R* = projected sales volume of supplies to facilities serving seniors and projected sales volume of home medical equipment respectively

The solution of this optimization problem is similar to our analysis of funeral requests. We assume that the independent pharmacist wants to sell the maximum volume of typical drugstore product lines. The exterior penalty function method continues to solve constrained optimizations with penalties for successive limits on prescription drugs, over-the-counter drugs, and retail merchandise where limits may be revised based on local demand. Bounds on senior supplies, home medical equipment are more predictable and may be held to binding limits. The exterior penalty function method imposes a penalty at each stage. For example, the limit on prescription drugs may be revised to M. The pharmacist would like to sell an unlimited amount of prescription drugs but is constrained by competition and availability. Suppose in the first stage, the decision-maker resets a preset limit on prescription drug sales to the more realistic limit of M. The preset limit may be presented as η so that the minimum cost function at stage 1 is

$\varphi(\eta) = |min(0,\eta)|\alpha$

and the difference between the older, higher limit η and the new limit γ has the following function (the limit only approaches M, thefinal limit, after all iterations have been completed,

$\zeta(\eta) = |\eta|\beta$

where α and β are given constants, with values of 1 or 2. A loss function results with with a revised limit at M after the first stage of penalty imposition. The loss function may be defined as,

$s(x) = \sum t =1m\varphi(x_1 -M \geq 0) + \zeta(\eta-\gamma)$

For any positive number *p* we may define the augmented objective function as

$F(x,p) = –[(r_1 – c_1)x_1 + (r_2 – c_2)x_2 + (r_3 – c_3)x3\ (r_4-c_4)x_4 +(r_5 – c_5)x_5 + (r_6-c_6)x_6]\ (1/p)[\ \sum t =1m\varphi(x_1 -M \geq 0) + \zeta(\eta-M)]$

A series of unconstrained optimizations are solved for$k = 0, 1, 2, 3, 4, 5$ given by

min $F(x, p_k) =–[(r_1 – c_1)x_1 + (r_2 – c_2)x_2 + (r_3 – c_3)x_3\ (r_4-c_4)x_4 +(r_5 – c_5)x_5 + (r_6-c_6)x_6]+ 1/p_k\{\sum m_i=1|min[0, x_1 + x_2 + x_3 + x_4 + x_5 + x_6 +x_7 \geq 0]|\alpha$

$\sum j=1p|\ x_1 , x_2 , x_3 , x_4 , x_5 , x_6 ,x_7,x_p = 0,1|\beta\}$ If x_k* is the optimal volume of prescription drugs, there will be a sequence of points (x_k) which converges to the optimal choice. From the cooperative's perspective, the profit may be defined as,

$$\Pi = (MF -CP – GA) – S = 0 \quad (19)$$

where

MF = membership fee

CP= cost of production including legal fees and wages to negotiators with wholesalers , vendors of retailmerchandise,

GA=general and administrative expenses including compensation for lobbyists with federal and state governments,

S =surplus returned to members.

3.7. Housing Cooperatives

A housing cooperative is a partnership between a corporation and a group of individuals formed for the purpose of facilitating home ownership for members. Members purchase certificates in the cooperative which permit them to own and occupy a single unit under a perpetual lease agreement (Sazama, 2000). For the individual decision-maker, the principal constraint is budgetary, with a strict limit on the monthly amount that the decision-maker may spend on housing. If current housing is a rental unit, the housing cooperative offers the social advantages of elimination of the outside landlord, lower crime, community control, building communities, shared maintenance, and the only opportunity for home ownership for members of certain socio-economic groups. Mobility, freedom from maintenance responsibilities and the freedom to make individual day-to-day housing-related decisions are some of the benefits of renting.

The binding budgetary constraint renders the condominium purchase unaffordable for this income segment. We state an objective function with the position of the decision-maker opting for the cooperative as x_1, and the rental choice

asx_2:

Max $-[LIx_1x_2 + CRx_1x_2, CCx_1x_2 + BCx_1x_2 + MEx_1x_2 + Sx_1x_2 + FFx_1x_2]$ (20)

St

$c_1x_1 + c_2x_2 \leq M$

$x_1 + x_2 = 0$

$x_1, x_2 = 0,1$

LI = freedom from the landlord , with 1= housing cooperatives, 0 = rentals,

CR = perceptions of reduced crime, with 1 = housing cooperatives, 0 = rentals,

CC = community control, with 1 = housing cooperatives, 0 = rentals,

BC = building communities, with 1 = housing cooperatives, 0 = rentals,

ME =mobility, with 1 = rentals, 0 = housing cooperatives,

S = freedom from shared maintenance, with 1 = rentals, 0 = housing cooperatives,

FF =freedom to make day-to-day non housing decisions, with 1 = rentals and 0 = housing cooperatives,

c_1 = cost of cooperative unit

c_2= cost of rental unit

M= individual budgetary allocation for housing

Since the Baumol revenue maximization model (Baumol, 1972), is being employed to achieve a solution, it is necessary to rewrite the above decision problem,

Max $-[LIx_1x_2 + CRx_1x_2 + CCx_1x_2 + BCx_1x_2 + MEx_1x_2 + Sx_1x_2 + FFx_1x_2]$(21)

St

$[c_1x_1 - c_2x_2] - [LIx_1x_2 + CRx_1x_2 + CCx_1x_2 + BCx_1x_2 + MEx_1x_2 + Sx_1x_2 + FFx_1x_2] - (x_1 + x_2) \geq -M$

$x_1 \geq 0 \quad x_1, x_2 = 0, 1$

$x_2 \geq 0$

The Lagrangian function becomes

$\Phi\ (LI, CR, CC, BC, ME, S, FF, x1, x2, M) = [LIx_1x_2 + CRx_1x_2, CCx_1x_2 + BCx_1x_2 + MEx_1x_2 + Sx_1x_2 + FFx_1x_2] + u(LIx_1x_2 + CRx_1x_2 + CCx_1x_2 + BCx_1x_2 + MEx_1x_2 + Sx_1x_2 + FFx_1x_2] - (c_1x_1 + c_2x_2) - (x_1 + x_2) - M$

Taking partial derivatives with respect to x1,

$\partial\varphi/\partial x_1 = \partial/\partial x_1\ [LIx_1x_2 + CRx_1x_2, CCx_1x_2 + BCx_1x_2 + MEx_1x_2 + Sx_1x_2 + FFx_1x_2] + u[\partial/\partial x_1(LIx_1x_2 + CRx_1x_2, CCx_1x_2 + BCx_1x_2 + MEx_1x_2 + Sx_1x_2 + FFx_1x_2) - -(c_1x_1 + c_2x_2) - (x_1 + x_2)] \leq 0$

$[LIx_2 + CRx_2 + BCx_2 + MEx_2 + Sx_2 + FFx_2] + u[(LIx_2 + CRx_2 + BCx_2 + MEx_2 + Sx_2 + FFx_2) - (c_1 + c_2x_2) - (x_2)] \leq 0$

Taking partial derivatives with respect to x_2,

$[LI + CR + BC + ME + S + FF] + u[(LI + CR + BC + ME + S + FF) - (c_1 + c_2)] \leq 0$

Rearranging terms yields the following Kuhn-Tucker condition,

$(1+u)[LI + CR + BC + ME + S + FF] - u(c_1 + c_2) \leq 0$ (22)

If $x_1, x_2 \geq 0$, the Kuhn-Tucker condition in (22) may be expressed as

$$\frac{\partial\,[LIx_1x_2+ CRx_1x_2, CCx_1x_2+BCx_1x_2 +MEx_1x_2 +Sx_1x_2 +FFx_1x_2]/\partial(x_1,x_2)}{\partial(-c_1x_1 +c_2x_2)/\partial(x_1, x_2)} =u/(1+u) \qquad (23)$$

Or

Marginal revenue/marginal cost $=u/(1+u)$ (24)

Since marginal revenue > 0, we conclude that $u > 0$, or that marginal revenue is positive but < marginal cost at the maximization of the objective function. This is a reasonable corollary given that the social benefits of membership in a housing cooperative may realize immediate benefit such as removal of the landlord and lower crime, while the social costs of less mobility and freedom in decision-making may be realized over a long period of time. For those who are committed to home ownership, the housing cooperative choice is one of two choices, i.e. between the housing cooperative or condominium ownership. The housing cooperative confers economic advantages as it gives an individual the right to own the assets of the housing corporation, affordable home ownership, no personal liability and cost savings in that only the equity of a departing member must be financed by an incoming member. The condominium offers an expanded array of housing choices and the freedom to make financing and maintenancedecisions within the unit. We state an objective function with the position of the decision-maker opting for the housing cooperative as x1, and the condominium choice as x_2:

$$\text{Max} -[BOx_1x_2+ MTx_1x_2+ PLx_1x_2+CS\,x_1x_2 + HCx_1x2 + F] \qquad (25)$$

St

$c_1x_1 +c_2x_2 \leq N$

$x_1 + x_2 = 0$

$x_1, x_2 = 0,1$

BO = building ownership , with 1= housing cooperatives, 0 = condominium ownership,

MT = mortgage terms, with 1 = housing cooperatives, 0 = condominium ownership,

PL= personal liability, with 1 = condominium ownership, 0 = housing cooperatives,

CS = cost savings on transfer of ownership, with 1 = housing cooperatives, 0 = condominium ownership,

HC =types of condominiums, 0 = housing cooperatives,

F = freedom to make maintenance and financing decisions, with 1 = condominium ownership, 0 = housing cooperatives,

c_1= cost of housing cooperative unit

c_2= cost of condominium unit

N= individual budgetary allocation for housing, N> M (budgetary limit for the rental choice-housing cooperative decision)

We employ the identical Baumol revenue maximization model (Baumol, 1972), as the rental versus housing cooperative decision, for which we rewrite the above decision problem,

$$\text{Max} -[BOx_1x_2+ MTx_1x_2, PLx_1x_2+CLx_1x_2 +HCx_1x_2\,] \qquad (26)$$

St

$[c1x1-c2x2]- [BOx1x2+ MTx1x2, PLx1x2+CLx1x2 +HCx1x2]-(x1 +x2)\geq -N$

$x1\geq 0 \quad x1,x2 = 0, 1$

$x2 \geq 0$

The Lagrangian function becomes

Φ $(BO, MT, PL, CL, HC, x_1, x_2, N) = [BOx_1x_2+ MTx_1x_2 +PLx_1x_2+CLx_1x_2 +HCx_1x_2] + u(BOx_1x_2+ MTx_1x_2, PLx_1x_2+CLx_1x_2 +HCx_1x_2] –(c_1x_1+c_2x_2) – (x_1 +x_2) -N$

Taking partial derivatives with respect to both $x_1$and x_2 and rearranging terms yields the following Kuhn-Tucker condition,

$$(1+u)[BO+MT+PL+CL+HC]-u(c_1+c_2 \leq 0 \tag{27}$$

If $x_1, x_2 \geq 0$, the Kuhn-Tucker condition in (26) may be expressed as

$$\frac{\partial\ [BOx_1x_2+ MTx_1x_2, PLx_1x_2+CLx_1x_2 +HCx_1x_2]/\partial(x_1,x_2)}{\partial(-c_1x_1 +c_2x_2)/\partial(x_1, x_2)} =u/(1+u) \tag{28}$$

Or

marginal revenue/marginal cost = $u/(1+u)$

since marginal revenue ≥ 0, we conclude that $u > 0$, or that marginal revenue is positive but < marginal cost at the maximization of the objective function as realize immediate benefits from cooperative membership may be realized such as easier mortgage qualification and lack of personal liability while the costs of expanded array of housing choices and freedom in decision-making may be realized over a long period of time. The profit of the housing cooperative would be the difference between revenue sources such as unit lease revenue(LE) and share certificate revenue (SR) and costs of maintenance (MA)and administrative expenses (GA). Dividends would be returned to shareholders.

$\Pi = (SR + LE – MA –GA) – Dividends = 0$

3.8. Mutual Insurance Companies

Mutual insurance companies resemble cooperatives in their ability to reduce property and casualty insurance costs for members which are often small businesses who lack the ability to negotiate low rates with large insurers. The Association of Mutual Insurance Cooperative offers property and casualty insurance to small businesses including bagel shops, bakeries, barber shops, beauty shops, book stores, restaurants, delicatessens, flower shops, graphic design firms, hardware stores,partially occupied mercantiles, pet groomers and pizzerias. Individual policies are directed towards single-family homeowners, condominium owners, seasonal owners, tenants and landlords.
If a small business owner is selecting from among five policies are represented as x1, x2, x3, x4, and x5, the cost of each policy is $c_1…c_5$, and the total budget is B, the following optimization model may be stated:

$$\text{Minimize } [c_1x_1 +c_2x_2+c_3x_3 +c_4x_4 +c_5x5] \tag{29}$$

Subject to

$[c_1x_1 +c_2x_2+c_3x_3 +c_4x_4 +c_5x_5]- B = 0$

$-x_1\leq 0$

$-x_2\leq 0$

$-x_3\leq 0$

$-x_4\leq 0$

$-x_5\leq 0$

At the minimum cost policy for that owner 1 choice, $x_1…x_5$ will be selected taking on the value of 1 and the rest will have values of 0. Any change in cost of selected policy due to reduction in the budgeted amount, B, may result in another policy assuming importance, and possiblybecoming the final choice.

3.9. Marketing Cooperatives

The cooperative's twin purposes are to reduce costs of production for mainstream products and provide publicity for specialty activities such as rentals and large-scale remodeling projects. Consequently, two objective functions may be combined for a hardware store member of the cooperative with the first one for the cost minimization of mainstream products contained in the cooperative's warehouses and the second one for revenue maximization from national advertising.

Minimize $(c_1x_1+c_2x_2\ldots+c_nx_n) - (a_1y_1+\ldots.a_1y_n+a_2z_1+\ldots.a_2z_n)$ (30)

Subject to

$c_1x_1+c_2x_2\ldots+c_nx_n \leq B$

$a_1y_1+\ldots.a_1y_n \leq D$

$a_2z_1+\ldots.a_2z_n \leq E$

$x_1, x_2, \ldots.x_n, y_1 \ldots y_n, z_1\ldots z_n \geq 0$

where,

$c_1\ldots c_n$ = cost per unit of mainstream products

$x_1\ldots.x_n$ = unit volume of hardware products purchased through the cooperative,

a_1 = advertising expenditure per unit for equipment rentals,

a_2……=advertising expenditure per unit for remodeling projects

B = budget limit for mainstream products,

D…….= budget limit for advertising for equipment rentals,

E = budget limit for advertising for remodeling projects.

Manufacturing or Technology Cooperatives

An engineering cooperative usually exists to provide custom-designed solutions to manufacturers to improve the efficiency of their production processes. Product lines include automated assembly, material handling, custom process, web handling and system integration (Isthmus Engineering, 2012). The function of the cooperative is social, rather than economic. If there are two engineers between whom there is a desire to promote interaction, the cooperative provides a forum for meeting attendance (*MA*), conflict resolution (*CR*), participation in voting (*PV*), dialog initiation (*DI*) and teamwork (*TW*), so that the following social profit maximization function may be stated,

Max $[MAx_1x_2+ CRx_1x_2+ PVx_1x_2+DIx_1x_2+ TWx_1x_2]$ (31)

St

$[MAx_1x_2+ CRx_1x_2, +PVx_1x_2+DIx_1x_2 + TWx_1x_2]-(x_1 +x_2)\geq -N$

$x_1\geq 0 \quad x_1,x_2 = 0, 1$

$x_2\geq 0$

The Lagrangian function becomes

Φ $(MA, CR, PV, DI, TW, x_1, x_2, N) = [MAx_1x_2+ CRx_1x_2, PVx_1x_2+DIx_1x_2 +TWx_1x_2] + u(MAx_1x_2+ CRx_1x_2, PVx_1x_2+DIx_1x_2 +TWx_1x_2]\geq -N$

Taking partial derivatives with respect to both $x_1$and x_2 and rearranging terms yields the following Kuhn-Tucker condition,

$(1+u)[MA+CR+PV+DI+TW] \leq 0$

Marginal revenue = *u*. As (*MA* + *CR* +*PV* +*DI* +*TW)*> 0, 1+*u*< 0, or *u*< -1 or *u* > +1 or *u*, the marginal revenue is positive and increasing. With anundifferentiated product, the cooperative shields small farmers from revenue losses byguaranteeingpurchases at a price at which production costs may be recovered and a small profit may be earned. The second form of service provided by cooperatives is that of to obtaining discounted inputs for businesses that would not be price competitive.In child care cooperatives, parental labor offsets some operating costs, credit unions make loans affordable and available to a larger number of people. Rural utilities are able to obtain loans that would not be approved by private banks, independent pharmacy cooperatives remain price competitive by purchasing discounted drugs and retail merchandise at prices negotiated by the cooperative, housing cooperatives provide an affordable path to home ownership, insurance cooperatives reduce insurance premiums for small businesses who may not qualify for preferential rates, and marketing cooperatives provide national advertising exposure and reduced prices for inputs. We have identified a third type of cooperative, i.e. the worker-owned manufacturing and technology cooperative which provides a forum for such social interaction, or makes an investment in the building of social capital.

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