

## The Optimal Allocation of Faculty across Colleges of Business

**Justin Lee Hughes**

School of Law  
University of Minnesota  
Minneapolis Minnesota  
United States of America

**Joseph P. McGarrity**

Professor of Economics  
University of Central Arkansas  
Conway, Arkansas 72035  
United States of America

### Abstract

*Administrators have little guidance from the academic literature when they attempt to spend resources efficiently. This paper seeks to help fill this need. It applies McGarrity's optimization model to show the optimal allocation of faculty lines across departments in Colleges of Business at five universities.*

### I. Introduction

University administrators must choose how to spend their budget dollars. Any effort by administrators to spend resources efficiently faces two hurdles. First, university politics can hamper these efforts. Harold (2000), Johnson and Turner (2009), and Ehrenberg (1999) suggest that departmental and university politics play a major role in determining how resources are allocated. Second, administrators who can overcome the political obstacles may still have trouble determining how to efficiently distribute their budget dollars. For instance, even though faculty salaries represent the largest component of university budgets, scholarly research has offered these administrators little guidance on how to efficiently spend the money devoted to faculty salaries.

To date, only one paper, McGarrity (2012), has developed a mathematically rigorous method on how to best allocate faculty positions.<sup>1</sup> He uses a Cobb Douglas utility function to determine the optimal number of tenured or tenured track faculty positions in each department in the College of Business at the University of Central Arkansas. McGarrity's model includes variables such as: the number of students taught, the number of majors, faculty salaries by department, salaries of recent graduates by major, as well as the college's budget.

This paper builds on McGarrity's research. It makes several contributions beyond those that appeared in McGarrity's paper. First, McGarrity only applied the model to one school; this paper applies the model to five schools: Virginia Tech, Purdue University, the University of Arkansas, George Mason University, and Texas Tech University. Second, in this paper, the recommended number of faculty slots for each department at a school is derived in part by using salary data for that department's graduates. McGarrity used salary data from a national survey, not from the specific school he was analyzing. Because workers are often paid the value of their marginal product, the productivity of students from this school is more closely linked to the salary data from that school than it is to salary data from national surveys.

### II. Data and Methodology

#### Methodology

We follow McGarrity (2012) and employ a Cobb Douglas utility function to find the optimal allocation of faculty. The equation in Lagrangian form is as follows:  $L = \max U(X_1^{a_1} X_2^{a_2} X_3^{a_3} X_4^{a_4} X_5^{a_5} X_6^{a_6} X_7^{a_7}) + \lambda(I - P_1X_1 - P_2X_2 - P_3X_3 - P_4X_4 - P_5X_5 - P_6X_6 - P_7X_7)$

$X_i$  represents the number of faculty in department  $i$ . In his paper, McGarrity applied this equation to the College of Business at the University of Central Arkansas, which has seven departments. Other Colleges of Business may have more or fewer departments than UCA. The equation will vary to accommodate the number of departments in a school.

$I$  is the budget spent on faculty salaries in the College of Business.  $P_i$  is a representative salary of a faculty member in department  $i$ . The parameter values  $a_i$  will sum to one.

To solve this utility function, take the natural log.

$$L = a_1 \ln X_1 + a_2 \ln X_2 + a_3 \ln X_3 + a_4 \ln X_4 + a_5 \ln X_5 + a_6 \ln X_6 + a_7 \ln X_7 + \lambda (I - P_1 X_1 - P_2 X_2 - P_3 X_3 - P_4 X_4 - P_5 X_5 - P_6 X_6 - P_7 X_7)$$

The first order conditions are:

$$\delta L / \delta x_i = a_i / x_i - \lambda P_i = 0$$

(for  $i=1 \dots 7$ )

Next, rearrange the first order conditions to solve for  $a_i$ .

$$a_1 = P_1 \lambda X_1$$

$$a_2 = P_2 \lambda X_2$$

and so on for all seven cases.

Next, we sum the left and the right side of the equation.

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 = \lambda (P_1 X_1 + P_2 X_2 + P_3 X_3 + P_4 X_4 + P_5 X_5 + P_6 X_6 + P_7 X_7)$$

The left side of the equation equals one because the parameters of any Cobb Douglas utility function sum to one. The part of the right side of the equation enclosed by a parenthesis must equal  $I$ , which is the budget for faculty salaries because we make the typical assumption that the entire budget is spent. The equation can be reduced to:

$$1 = \lambda I$$

Therefore,  $\lambda = 1/I$

$$\text{Remember that } a_i = P_i \lambda X_i$$

Solving for  $X_i$

$$X_i = a_i / \lambda P_i$$

We can plug in  $1/I$  for  $\lambda$  to get

$$X_i = a_i I / P_i$$

This gives the optimal number of faculty for each department. It shows the optimal number of faculty members in a department is determined by  $a_i$ , the relative weight faculty members in this discipline give to a college's utility function, by  $I$ , the college wide salary budget, and  $P_i$ , the salary of the faculty in that department.

The parameters  $a_i$ , which represents the relative importance of faculty in each department, deserves more discussion. It captures how much each department adds to society's welfare, which is measured by the graduating students' starting salaries, multiplied by a measure of the number of graduates from each department. In a competitive labor market, the value of the marginal product of labor equals the wage rate, meaning that a firm will hire additional workers until the value of the work by an additional employee is equal the wage rate. The value of the marginal product of labor curve constitutes the firm's demand curve for labor. Thus, it is a good indicator of the value the graduate is adding to society during their first year out of school. Of course, we cannot observe the demand curve for labor. However, we can observe the wage rate which will be a useful proxy for the value of the marginal product. Each school's output of majors can be considered the last workers hired and because each school supplies such a small fraction of the total supply of workers in that major, the value of the marginal product and the wage rate will be very similar.

However, the university is not responsible for all of the workers value in the workplace. Previous education must be taken into account. To determine the value added by a university, we must look at the value added before the student arrived at college, and subtract that from their first year wage-rate. We used the latest data from NCES (National Center for Education Statistics) to determine how much value a high school diploma adds to a person.

### Parameter values

In order to aggregate the enrollment data so that it can be matched up with the salary data, we take several approaches. First, we calculate how many students each department could graduate in a given year if students only took courses in that department. To make that calculation, we take the number of credit hours needed to graduate at a given institution, and divide that by 3, which is the typical number of credit hours per class. All of the schools in this study, save for one, had a credit hour requirement of 120 to graduate.<sup>1</sup> We divide 120 by 3 in order to get 40. This is the number of courses a student has to take in order to graduate. The next step is to divide the number of students by 40 (or the equivalent for other school). For example, if the 944 accounting students at the University of Arkansas only took accounting classes, the accounting faculty could teach enough classes so that 23.6 students could graduate. We call this the Full Time Equivalent.

Our next step is to put a value on the student's education. We use the starting salaries of each discipline in each school as a measure of value for the education that the student's received at each institution. Graduate salary surveys were obtained for the various schools, and the averages for each discipline were used. To calculate the total value for each discipline in each school, we multiply the number of Full Time Equivalent students by the difference in salaries between college and high school students. To calculate  $a_i$ , we take the total value of a discipline and divide that by the total value of the college as a whole.

### Solving for the optimal allocation of faculty

We will solve for the optimal allocation of faculty by using the faculty wage-rates provided by colligatetimes.com, as well as from various newspapers that have databases of public salaries.<sup>2</sup> The salaries on these websites were obtained through Freedom of Information Act (FOIA) requests. The information listed was crosschecked with the online directory that each University provides of their faculty. If a faculty member was listed on a school's website, but not found in one of the online newspaper databases, or in colligate times, then he or she was left out of the study, which is operating under the assumption that the FIOA requests are more reliable than the information posted of any particular school's website. Salary data for the last three years was looked at in order to determine if and when a faculty member left an institution. To calculate the optimal allocation of faculty, we use the total budget each college has for its faculty, the  $a_i$ s, as well as the average salary of the faculty in each discipline.

We calculate the optimal allocation of faculty using the following equation:  $X_i = A_i / P_i$ .  $I$  is the total amount spent on faculty,  $a_i$ , and  $P_i$ , the average faculty salary for the faculty in each discipline.

### III. Analysis

For all five schools, Table 1 displays the actual number of tenure-track faculty slots (ACT), the optimal number of tenure track slots (OPT), and the difference between the actual and the optimal (DIFF). A negative number in the DIFF column indicates how many actual TENURE TRACK slots need to be reduced in order to reach a department to reach its optimal level. Likewise, a positive number indicates how many TENURE TRACK slots need to be added to reach the optimal level.

Generally, Accounting is understaffed. Four of the five schools should hire more accounting faculty. George Mason University should hire the most new accounting faculty. The exception, the University of Arkansas, has too many accounting faculty, but they are very close to being optimally staffed. They are only overstaffed by only one faculty line. In contrast, MIS is quite overstaffed. All four schools with an MIS department should reduce their number of MIS faculty lines. The universities that had the most and the least changes to make in their allocation of accounting faculty also had the most and least changes that were needed in their MIS allocation. George Mason University has the largest change to make to bring their numbers in line with their optimal allocation of MIS faculty. The University of Arkansas has MIS staffing that is very close to its optimal level of staffing.

---

<sup>1</sup> Purdue has a 126 credit hour requirement

<sup>2</sup> IndyStar.com, startribune.com, arkansasonline.com, texastribune.org,

Table 1: Actual and Optimal Tenure Track Slots

School	Acct			Mktg			Mgmt			Finance			Econ			MIS			Hop		
	ACT	OPT	DIFF	ACT	OPT	DIFF	ACT	OPT	DIFF	ACT	OPT	DIFF	ACT	OPT	DIFF	ACT	OPT	DIFF	ACT	OPT	DIFF
Virginia Tech	22	24.9	2.9	12	11.6	-0.4	16	17.5	1.5	18	24.7	6.7	13	9.4	-3.6	20	17.4	-2.6	11	6.4	-4.6
GMU	17	30.7	13.7	12	11	-1	16	7.4	-8.6	10	13.2	3.2				15	3.87	-11.13			
U of Ark	10	8.8	-1.2	10	12.6	2.6	18	16.3	-1.7	11	11.4	0.04	16	17.7	1.7	7	4.9	-2.1			
Purdue	7	16.8	9.8				43	37.5	-5.5				20	13.7	-6.3						
Texas Tech	12	15	3	8	14.2	6.2	9	14.8	5.8	10	10.2	0.2				15	4.6	-10.4			

However, in order to make a meaningful comparison across schools, we must take the differences in the size of the schools into account. We would expect a large school to have to make large adjustments in order to reach their efficient allocation of faculty lines, while a small school could make small adjustments. Table 2 gives the number of tenure track slots in the whole college of business in each of the five schools. Virginia Tech employs the most faculty and Texas Tech employs the fewest professors. The other three universities are roughly the same size.

Table 2: Percentage Deviation from Optimal Allocation

Schools	Actual TT	ABS of Diff	Percentage
Virginia Tech	112	22.3	19.9%
GMU	70	37.6	53.7%
U of Ark.	72	9.7	13.5%
Purdue	70	21.6	30.9%
Texas Tech	54	25.6	47.4%

The next column in Table 2 is a measure of the number of faculty lines each university differs from their optimal distribution of lines within a college of business. These numbers are derived by using the information listed in the DIFF column in Table 1. Remember, this column showed the optimal number of faculty lines in a department minus the actual number of faculty lines in that department. Because these numbers are both negative and positive, we take the absolute values of these differences. Next we sum these absolute values for each college. These sums are reported in the third column of Table 2. In the final column, we report the percentage of faculty slots that are misallocated. We obtain this figure by dividing column 3 by column 2. These results show that the University of Arkansas has the best allocation of faculty slots, while George Mason University has the worst. However, generally, these schools could all benefit by reallocating their faculty lines.

The use of instructors, Professors of Practice, Adjunct faculty, and TA’s may help us understand the deviations that we are seeing. Listed below in Table 3 is the total number of non-tenure track teachers in each department. The department that is most underrepresented in Table 1, Accounting, hires the most instructors. Strangely though, the most overrepresented department (MIS) hires the second most instructors.

Table 3: The Use of Instructors

Schools	Accounting	Marketing	Management	Finance	Economics	MIS
Virginia Tech	6	5	4	6	8	2
GMU	28	10	11	8		29
Purdue	9	5		7	12	5
Texas Tech	6	2	12	4		4
U of Ark	4	6	9	5	3	7
ABS	54	28	36	30	23	47

The data suggests the use of instructors alone does not explain why one school is more efficient than another. Arkansas is very close to its most efficient allocation of faculty lines, yet it uses a large amount of instructors (34).

#### **IV. Conclusion**

This paper has examined five colleges of business from around the country using a constrained Cobb Douglas utility function to solve for the optimal allocation of faculty at five universities: the University of Arkansas, Virginia Tech, George Mason, Purdue, and Texas Tech. We discovered that several things:

- Accounting is a department that is consistently (and severely) understaffed in the colleges we studied, while MIS is consistently overstaffed.
- The University of Arkansas allocates its faculty relatively well and George Mason University does so relatively poorly.

As more and more families look for cheaper options for their children's education, administrators will have less leeway to increase tuition in order to solve their budgetary problems. They will have to face the political reality of transferring, adding, or reducing faculty lines in certain departments. This paper should lessen budgetary problems by helping administrators use their resources more efficiently.

#### **References**

- Becker, W. G. (2010). Do undergraduate Majors or PH.D. Students Affect Faculty Size.
- Clark, K. (2010, July 12). Most Public Colleges Face Budget Cut Threats in 2011. Retrieved March 30, 2012, from [www.usnews.com: http://www.usnews.com/education/articles/2010/07/12/most-public-colleges-face-budget-cut-threats-in-2011](http://www.usnews.com/education/articles/2010/07/12/most-public-colleges-face-budget-cut-threats-in-2011)
- Ehrenberg, R. G. (1999). Adam Smith goes to college: an economist becomes an academic administrator. *Journal of Economic Perspectives*, 1 (13), 99-116.
- Ehrenberg, R. G. (n.d.). Studying Ourselves: The Academic Labor Market. Retrieved October 25, 2011, from <http://digitalcommons.ilr.cornell.edu/workingpapers/31>.
- Harold, T. (2000). Power in the Resource Allocation Process: the impact of 'rational' systems. *Journal of Higher Education Policy and Management*, 22 (2).
- McGarrity, J. (2012). The Optimal Allocation of Faculty in a College of Business. *International Journal of Business and Social Science*, 3,12, June

#### **Notes**

---

<sup>1</sup> While not attempting to find an optimal allocation of faculty lines, Becker, Greene, and Siegfried attempt to explain the link between the number of faculty lines and student enrollment. They find the number of students taught by one department influences that department's size. In schools without a graduate program, it takes an increase of 26 or 27 undergraduate economics bachelor degrees in a moving three year average to add one more faculty position (Becker). Over longer periods of time when an increase in enrollment is considered to be permanent, it takes an additional of 9 or 10 students to change the faculty size by one. In doctoral-granting institutions, undergraduate degrees have no effect on faculty size. However, an increase in just one doctoral student in the long term can add another faculty line.